Interpolation of TW time transfer from measured points onto standard MJD for *UTC* generation

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Abstract

We often need to determine the time transfer result for an epoch at which no direct measurement was made. For example, the time tag used in *Circular T* is 0 h *UTC* of the standard MJD, which is not generally a measuring epoch. Usual time transfer practice is to first smooth the raw data to filter out white measurement noise, and then interpolate the smoothed data to determine the time transfer value for the required epoch. However, up until the end of 2009, for two-way (TW) time transfer only simple linear interpolation (but any high-order interpolation method) was used. This was because, when TW was introduced into the computation of *UTC* in 1998, there were not enough measured points to allow a mathematically rigorous smoothing-interpolation process. Since the end of 2005 the number of measurements has increased to between 12 and 24 per day. To exploit the redundancy of this increased number of data points, in 2005 it was proposed that a higher order smoothing-interpolation method be used. It was concluded that: (1) a high-order smoothing interpolation is better than a simple linear interpolation; and (2) Vondrak smoothing is preferable to other high-order smoothing-interpolation techniques.

This paper builds on the previous study, examining the Vondrak smoothing-interpolation method with the benefit of a new powerful tool: the GPS PPP time-transfer solution. This provides an objective measure of the performance of different smoothing-interpolation methods. We conclude that in most cases Vondrak smoothing to the power 10^5 leads to the best interpolation results.

1. Introduction

In practice, we often need to determine the time transfer value at a particular epoch for which there was no direct measurement. The time tag used in *Circular T* is 0 h *UTC* of the standard MJD, which is not generally a measured point.

In GPS time transfer, there are about 100 measured epochs per day. The raw data are first smoothed to filter out the white measurement noise and then the smoothed data are used to interpolate the time transfer values at the required epochs. In *UTC* generation, Vondrak smoothing is used for GPS data and the power of the filtering is chosen to reduce the time transfer noise but maintain the clock behaviors. The smoothing should not reduce or remove any biases in the raw data. For TW time transfer, only linear interpolation is used but any high-order interpolation method. Originally this was because there were not enough measuring points to allow a mathematically rigorous smoothing-interpolation. In fact, before 2004 there were only 3 measurements per week; in 2004 and 2005 the number of measured points increased to 4 per day; and since the end of 2005, there have been 12 to 24 points per day. Use of a high-order smoothing-interpolation method thus becomes possible. In a linear interpolation only two points are used: the point just before midnight and the point just after midnight. None of the other measured points are considered and obviously any measurement error in the two points is directly transferred in the interpolation.

To fully exploit the potential in more numerous redundant measurements, in 2005, use a high-order smoothing-interpolation method was proposed in [1], which compared the following 9 smoothing-interpolation techniques:

- linear
- rational
- polynomial
- splint
- least square linear fitting
- least square polynomial fitting
- least absolute residual fitting
- weighted mean
- Vondrak smoothing

It was concluded [1] that: (1) when there are more than 12 points per day, high-order smoothing-interpolation is better than simple linear interpolation; and (2) Vondrak smoothing seems to be better than other high-order smoothing-interpolation techniques. Unfortunately, this study was not continued because no independent time-linking technique as precise as TW was available to evaluate the improvement due to higher order techniques.

GPS PPP has been operational as an accurate time-transfer technique since 2008 and is now used in *UTC* time transfer. It has the advantages of short-term stability. The measurement noise is expected to be about 100 ps or less. The Type A uncertainty (u_A) of GPS PPP is of the order of 0.3 ns [3,4]. In addition, GPS PPP is a completely independent observation from TW and is such an ideal tool to evaluate the performance of different smoothing-interpolation techniques.

In this paper, we continue the study of [1] with the help of GPS PPP and concentrate on the Vondrak smoothing-interpolation.

We address the following questions:

- Will the white measurement noise in TW time transfer be filtered?
- Is the final interpolation result for *UTC* computation improved?
- What is the most suitable power for the Vondrak smoothing?

2. The method

The goal of this study is to reduce the white noise in the time link measurements. The following points were taken into account in evaluating the performance of the different methods or the different parameters used in each method:

- Comparison of the triangle closures of the TW network obtained, cf. Table 1. As a triangle finite closure indicates a true error, it provides a reference to identify the improvements. However, because the triangle closure is the true but the total error in the measurements, it should not be used as the only criterion.
- Comparison of the differences between the GPS PPP and TW links obtained, cf. Table 2 and 3. As GPS PPP is a low-noise and independent technique, it is an ideal reference to identify the gain in measurement noise filtering. Comparison with GPS PPP is used as the major criterion in this study.
- The aim is to reduce the white noises in the TW time transfer. Diurnal variations, although they are the main source of error in TW time transfer, should in principle not be filtered in the smoothing. Therefore, the wavelength of the filtering should not be longer than half day (12 to 24 points per day). This implies that the power of the Vondrak smoothing should not be much less than 10^5.
- Finally, we compared interpolated values with measured values. Although not presented here, the result was similar to the above studies.

In the following section, we discuss the results of numerical tests and analysis.

3. Results of the numerical tests and discussion

The test data set is that of the official TW and GPS PPP time links used for the computations of *UTC* in July and August 2009 (data sets 0907 and 0908, corresponding to *Circular T Nos*. 259 and 260 [2] between MJD 55014-55039 and 55044-55074). The distances of the time links range from 100 km to nearly 10 000 km; with short baselines between the European laboratories, long baselines between Europe and America, and very long baselines between Europe and Asia.

We compare the triangle closures and the discrepancies between TW and GPS PPP obtained using linear or Vondrak (Vdk) smoothing (the latter to different powers). Obviously, smaller values of the closures and discrepancies indicate a better method or parameter setting.

A comparison of the mean values of the triangle closures (Tab. 1a) and the mean values of the differences (Tab. 2a, 3a) between TW and GPS PPP shows that the smoothing-interpolated values are almost unchanged, i.e., the estimation is non-deviated. This is important and implies that the mathematical expectation of the Vondrak smoothing approach is the same as that of the raw data.

Both the triangle closures (Tab. 1a and 1b) and the comparison to GPS PPP (Tab. 2a, 2b and 3a, 3b) demonstrate that statistically Vondrak smoothing to power 10^9 gives a result no better than that of the linear interpolation. This defines the upper limit of the Vondrak smoothing power.

The triangle closures and the discrepancies between TW and GPS PPP links were compared for Vondrak powers in the range from 10^0 to 10^9. In order to minimize the triangle closures and the discrepancies, a Vondrak smoothing power of between 10^4 to 10^6 seems reasonable. As our goal is to reduce only the white noise in the measurements without affecting the biases, e.g. the diurnals, we verified the diurnal signals after Vondrak smoothing of power 10^4, 10^7 and 10^9 (see the bottom-right plots in Fig. 2 to Fig. 4). In each case they remain the same as in the raw links (Fig. 1). Such, if there are enough data points and the power of the Vondrak smoothing is reasonable, the white measurement noise can be considerably reduced without deformation of the raw data.

We stress that minimizing the triangle closure is not the only criterion used to determine the optimal Vondrak power. In Table 1b for example, the Vondrak smoothing to power 10^1 gives in most case the smallest closure. However, as mentioned above, it might be too strong to filter only the TW transfer measurement noises and it might affect certain biases in the links and even the clock behaviors.

Generally speaking a more stable TW link requires a higher power of Vondrak smoothing; for example 10^6 or 10^7 is more suitable for NIST-PTB while 10^4 or 10^5 is more suitable for NICT-PTB, which is noisier.

In conclusion, depending on the purpose of application and the quality of the link, Vondrak smoothing to 10^5 or 10^6 is the best filter in most cases.

Table 1a Comparison of the mean triangle closures computed with linear interpolation and Vondrak smoothing-interpolation to different powers. No. is the number of data point.

			Mean triangle closures / ns					
Triangles	**No.**	**Lin.**	**Vdk1**	**Vdk3**	**Vdk5**	**Vdk6**	**Vdk7**	**Vdk9**
PTB CH IT	194	-0.10	-0.12	-0.11	-0.10	-0.10	-0.10	-0.10
PTB CH NIST	169	-0.41	-0.45	-0.43	-0.42	-0.42	-0.41	-0.41
PTB CH OP	167	-0.21	-0.17	-0.17	-0.20	-0.21	-0.21	-0.21
PTB CH SP	168	-0.15	-0.18	-0.16	-0.15	-0.15	-0.15	-0.15
PTB CH USNO	162	0.08	0.09	0.08	0.08	0.08	0.08	0.08
PTB CH VSL	121	-0.28	-0.27	-0.25	-0.26	-0.26	-0.27	-0.28
PTB IT NIST	173	-0.18	-0.16	-0.17	-0.18	-0.18	-0.18	-0.18
PTB IT OP	166	-0.26	-0.27	-0.26	-0.26	-0.26	-0.26	-0.26
PTB IT SP	167	-0.08	-0.09	-0.08	-0.08	-0.08	-0.08	-0.08
PTB IT USNO	170	-0.64	-0.61	-0.63	-0.64	-0.63	-0.64	-0.64
PTB IT VSL	128	-0.33	-0.35	-0.36	-0.34	-0.33	-0.33	-0.33
PTB NIST OP	170	0.27	0.28	0.29	0.27	0.26	0.26	0.27
PTB NIST SP	171	0.40	0.39	0.40	0.40	0.40	0.40	0.40
PTB NIST VSL	140	0.13	0.08	0.09	0.14	0.14	0.13	0.13
PTB OP SP	171	0.13	0.11	0.11	0.13	0.14	0.13	0.13
PTB OP USNO	166	0.16	0.21	0.17	0.17	0.17	0.17	0.16
PTB OP VSL	128	0.03	-0.12	-0.06	0.02	0.02	0.02	0.03
PTB SP USNO	165	0.21	0.26	0.23	0.22	0.22	0.22	0.21
PTB SP VSL	128	0.05	0.01	0.01	0.04	0.05	0.05	0.05

Table 1b Comparison of the standard deviations between the triangle closures computed with linear and Vondrak smoothing-interpolation to different powers

			Standard deviations / ns					
Triangles	**No.**	**Lin.**	**Vdk1**	**Vdk3**	**Vdk5**	**Vdk6**	**Vdk7**	**Vdk9**
PTB CH IT	194	0.40	0.19	0.22	0.28	0.32	0.36	0.40
PTB CH NIST	169	0.26	0.14	0.16	0.18	0.20	0.22	0.25
PTB CH OP	167	0.53	0.17	0.30	0.37	0.43	0.47	0.53
PTB CH SP	168	0.55	0.38	0.43	0.46	0.49	0.52	0.55
PTB CH USNO	162	0.40	0.27	0.29	0.33	0.35	0.37	0.40
PTB CH VSL	121	1.14	0.50	0.60	0.88	1.02	1.08	1.13
PTB IT NIST	173	0.47	0.33	0.35	0.39	0.40	0.43	0.47
PTB IT OP	166	0.47	0.32	0.38	0.38	0.40	0.43	0.47
PTB IT SP	167	0.41	0.29	0.31	0.35	0.36	0.38	0.41
PTB IT USNO	170	0.78	0.58	0.64	0.69	0.72	0.75	0.77
PTB IT VSL	128	0.54	0.20	0.32	0.42	0.44	0.47	0.53
PTB NIST OP	170	0.43	0.13	0.17	0.27	0.34	0.38	0.42
PTB NIST SP	171	0.47	0.31	0.33	0.38	0.42	0.44	0.47
PTB NIST VSL	140	0.87	0.33	0.53	0.67	0.78	0.83	0.87
PTB OP SP	171	0.38	0.19	0.23	0.27	0.30	0.33	0.38
PTB OP USNO	166	0.43	0.17	0.28	0.31	0.35	0.38	0.42
PTB OP VSL	128	0.66	0.28	0.41	0.47	0.54	0.58	0.65
PTB SP USNO	165	0.54	0.35	0.41	0.45	0.48	0.51	0.54
PTB SP VSL	128	0.64	0.24	0.39	0.43	0.48	0.54	0.63

Table 2a Comparison of the differences between the GPS PPP and TW links computed with linear and Vondrak interpolations (0907 Mjd 55014-55039)

			Difference of GPS PPP and TW / ns					
Baseline	**No.**	**Linear**	**Vdk0**	**Vdk1**	**Vdk3**	**Vdk5**	**Vdk7**	**Vdk9**
USNO-PTB	293	-3.58	-3.57	-3.57	-3.58	-3.58	-3.58	-3.58
USNO-PTB	293	-2.70	-2.69	-2.69	-2.70	-2.70	-2.70	-2.70
NIST-PTB	370	-1.20	-1.19	-1.19	-1.20	-1.20	-1.20	-1.20
NPL-PTB	354	1.27	1.28	1.28	1.27	1.27	1.27	1.27
NICT-PTB	678	-10.88	-10.88	-10.88	-10.88	-10.88	-10.88	-10.88

Table 2b Comparison of the standard deviations of the differences between the GPS PPP and TW links computed with linear and Vondrak interpolations (0907 Mjd 55014-55039)

			Standard Deviation / ns							
Baseline	**No.**	**Linear**	**Vdk0**	**Vdk1**	**Vdk3**	**Vdk4**	**Vdk5**	**Vdk7**	**Vdk8**	**Vdk9**
USNO-PTB	293	.530	.606	.548	.476	**.473**	.487	.514	.520	.527
USNO-PTB	293	.412	.556	.479	.367	**.353**	.369	.402	.404	.410
NIST-PTB	370	.294	.584	.479	.349	.322	.300	.289	**.286**	.291
NPL-PTB	354	.414	.608	.500	.385	.363	**.348**	.356	.377	.405
NICT-PTB	678	.991	.713	.591	**.527**	.545	.691	.895	.926	.950

Table 3a Comparison of the differences between the GPS PPP and TW links computed with linear and Vondrak interpolations (0908 Mjd 55044-55074)

			Difference of GPS PPP and TW / ns				
Baseline	**No.**	**Linear**	**Vdk1**	**Vdk3**	**Vdk5**	**Vdk7**	**Vdk9**
USNO-PTB	382	-2.999	-3.004	-2.999	-2.999	-2.999	-2.999
NIST-PTB	415	-1.619	-1.619	-1.619	-1.619	-1.619	-1.619
NICT-PTB	709	-5.702	-5.706	-5.702	-5.702	-5.702	-5.702

Table 3b Comparison of the standard deviations of the differences between the GPS PPP and TW links computed with linear and Vondrak interpolations (0908 Mjd 55044-55074)

			Standard Deviation / ns				
Baseline	**No.**	**Linear**	**Vdk1**	**Vdk3**	**Vdk5**	**Vdk7**	**Vdk9**
USNO-PTB	382	0.714	0.628	**0.533**	0.555	0.640	0.706
NIST-PTB	415	0.334	0.508	0.379	0.325	**0.310**	0.329
NICT-PTB	709	1.036	0.491	**0.471**	0.687	0.909	0.994

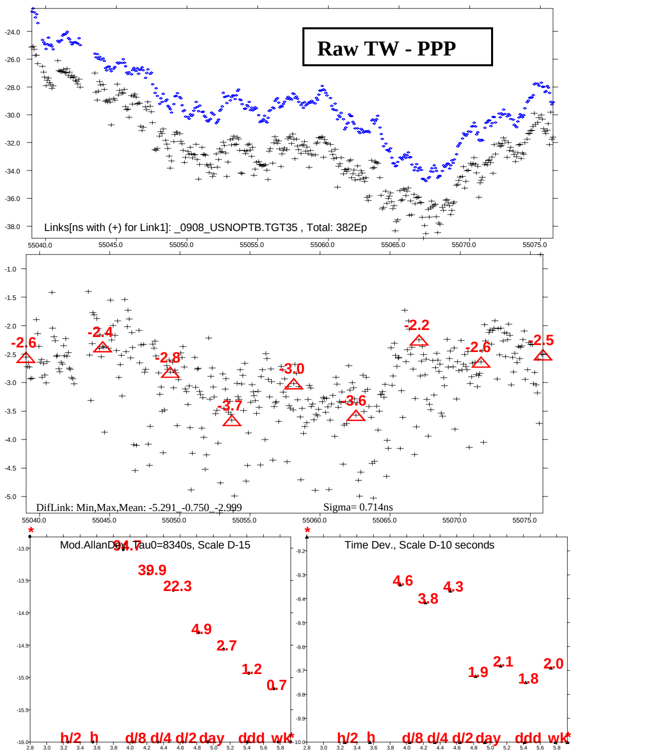

Fig. 1 Time link comparison between GPS PPP and TW with linear interpolation on the baseline USNO-PTB 0908 / ns

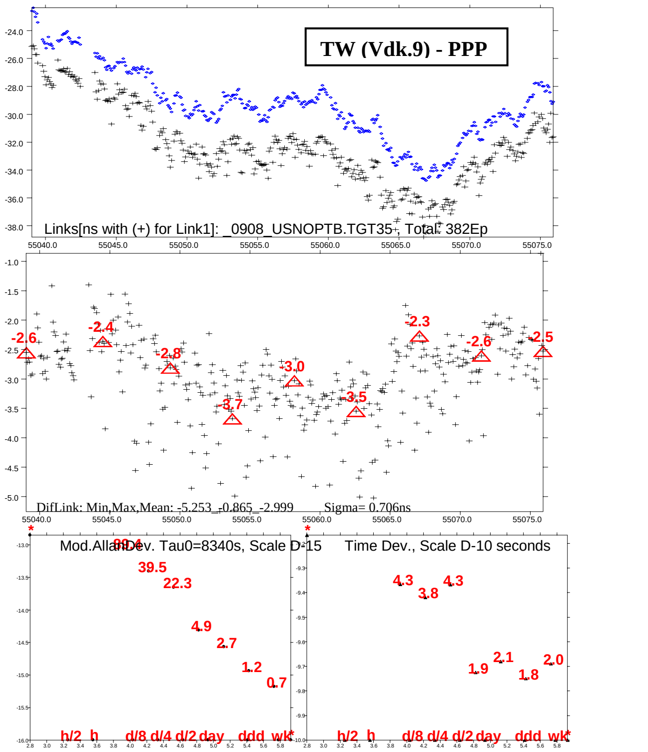

Fig. 2 Time link comparison between GPS PPP and TW with Vondrak power 10^9 smoothing-interpolation on the baseline USNO-PTB 0908 / ns

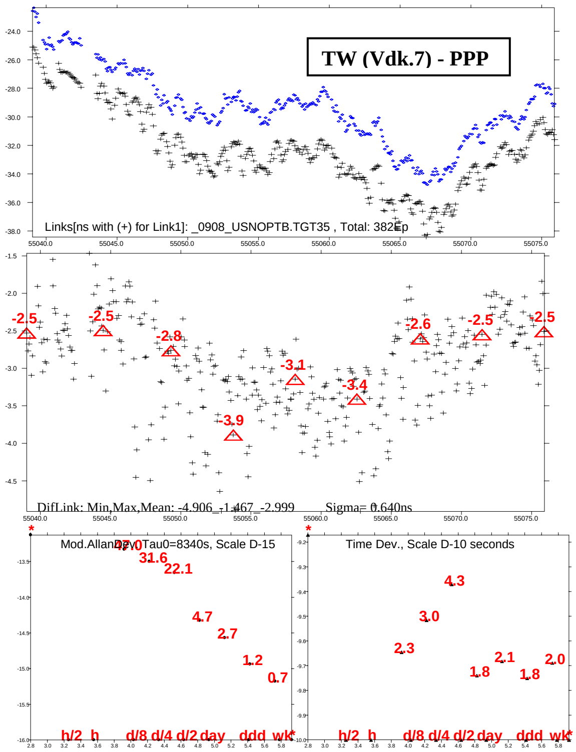

Fig. 3 Time link comparison between GPS PPP and TW with Vondrak power 10^7 smoothing-interpolation on the baseline USNO-PTB 0908 / ns

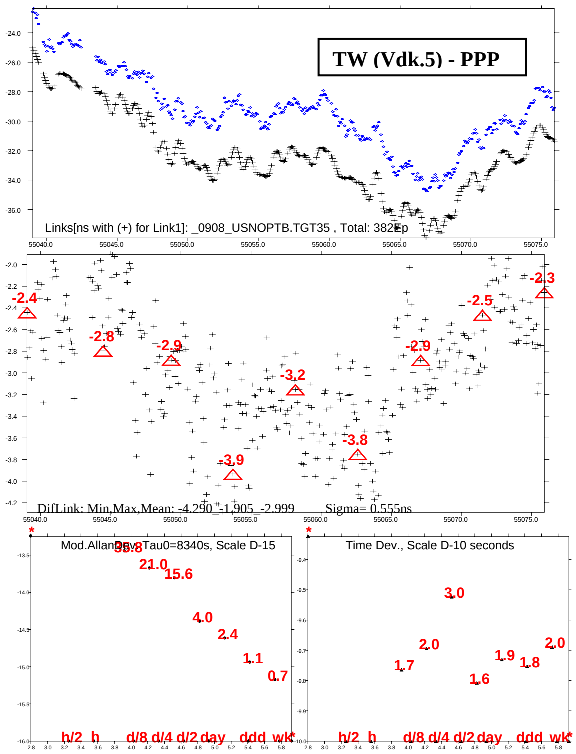

Fig. 4 Time link comparison between GPS PPP and TW with Vondrak power 10^5 smoothing-interpolation on the baseline USNO-PTB 0908 / ns

4. Conclusion

Building on an earlier study [1] with the help of the recently operational GPS PPP technique, we re-studied the smoothing-interpolation technique for TW time transfer data to be used in *UTC* generation.

Triangle closures of the TW network and the discrepancies between TW and GPS PPP results were compared. The smoothing-interpolation technique which gives the smallest closure and discrepancy is the best one.

This work supports the conclusion of the earlier study that high-order smoothing-interpolation is better than a simple linear interpolation, and the Vondrak smoothing gives a better result than other high-order smoothing-interpolation techniques.

We conclude that, in most cases, Vondrak smoothing-interpolation with a power 10^5 gives the best result. The method has therefore been installed in the *UTC/TAI* computation software package Tsoft since for the monthly *Circular T* computation.

References:

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[2] BIPM *Circular T 259*, July 2009, Section 6, http://www.bipm.org/jsp/en/TimeFtp.jsp?TypePub=publication

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[4] BIPM *Circular T 266*, Mars 2010, Section 6, http://www.bipm.org/jsp/en/TimeFtp.jsp?TypePub=publication